Key

Additional Problems: Oxygen Transport

1) \[ Y = \frac{P_{O_2}^n}{P_{O_2}^n + P_{CO_2}} \]
   \[ n = 1, P_{CO_2} = 17 \text{ torr}, \quad P_{O_2} = 2.43 \text{ torr} \]
   \[ Y_{res} = \frac{2.43}{2.43 + 17} = 0.08 \]
   \[ \text{or } P_{O_2} = 5.5 \text{ torr} \]
   \[ Y = \frac{5.5}{5.5 + 17} = 0.25 \]

2) \[ n = 2.8, P_{O_2} = 26 \text{ torr}, \quad P_{CO_2} = 15.0 \text{ torr} \]
   \[ Y = \frac{(15.0)^{2.8}}{(15.0)^{2.8} + (26)^{2.8}} \]
   \[ Y = \frac{(15.0)^{2.8}}{(15.0)^{2.8} + (26)^{2.8}} \]
   \[ Y_{res} = 0.18 \]
   \[ Y = 0.97 \]

3) a) \[ \Delta Y = Y_{tissue} - Y_{tissue} = \frac{100^{2.8}}{100^{2.8} + 26^{2.8}} - \frac{20^{2.8}}{20^{2.8} + 26^{2.8}} \]
   \[ \Delta Y = 0.98 - 0.32 = 0.66 \]

b) 100 molecules of Hb is 400 subunits or 400 O₂-binding sites. On average, 58% of these sites will fill in the lung, but only 32% will remain filled after passing through tissues.

So the capacity to deliver O₂ is 66% of the number of carrier sites: 400 x 0.66 = 264 O₂ molecules 264 molecules O₂ delivered per 100 molecules Hb in hemoglobin.

4) a) Non-cooperativity \[ n = 1 \]
   \[ \Delta Y = \frac{100^{2.8}}{100^{2.8} + 26^{2.8}} - \frac{20^{2.8}}{20^{2.8} + 26^{2.8}} = 0.35 \]

b) Mutations at subunit interfaces might affect cooperativity. But because of the complexity of the cooperative mechanism, it’s hard to rule out other sites. Probably not surface residues on hemoglobin remain...
5. a) \( n = 2.4 \quad P_{50} = 10 \text{ torr} \)

\[
\Delta V = \frac{100^2 \times 10^2}{100^2 + 10^2} = \frac{20^2}{20^2 + 10^2} = 0.99 - 0.77 = 0.12
\]

(Transport reduced by about 80%)

b) Malatrace & BP6 binding site might lower affinity for BP6, thus lowering P50.

6. See next page
6. a) \[ Y_A = \frac{a}{a+P_{50}} \quad Y_b = \frac{b}{b+P_{50}} \]

Let \( P = \sigma P_{50} \) (to save writing)

\[ \Delta Y = \frac{a}{a+\sigma P_{50}} - \frac{b}{b+\sigma P_{50}} = \frac{a(a+\sigma P_{50})-b(a+\sigma P_{50})}{(a+\sigma P_{50})(b+\sigma P_{50})} \]

Now, simplify.

\[ \Delta Y = \frac{a\sigma P_{50} + a^2 - b\sigma P_{50} - b^2}{(a+b)(a+\sigma P_{50})(b+\sigma P_{50})} = \frac{a(b-a)}{(a+b)(a+\sigma P_{50})(b+\sigma P_{50})} \]

Take derivative with respect to \( P \) (\( a \neq b \) and \( b \neq \sigma P_{50} \))

\[ \frac{d}{dP} \Delta Y = \left( P^2 + P(a+b) + ab \right) \left( a-b \right) - P(a-b) \left( 2P + a+b \right) \]

\[ \frac{d}{dP} \Delta Y = \frac{1}{(P^2 + P(a+b) + ab)^2} \]

If \( \frac{a}{b} = 0 \) then \( x = 0 \), so \( \frac{d}{dP} \Delta Y = 0 \) if the numerator = 0.

\[ (a-b) \left[ P^2 + P(a+b) + ab - 2P^2 - aP - bP \right] = 0 \]

If \( a \neq b \), then \( a-b \neq 0 \), so denominator becomes 0. (If \( a = b \), no need to consider, since no transport)

\[ P^2 + P(a+b) + ab - 2P^2 - P(a+b) = 0 \]

or \[ -P^2 + ab = 0 \]

or \[ P^2 = ab \]

b) For \( 100 \text{ ton to 20 ton} \), \( a = 100, \ b = 20 \),

\[ P = \sqrt{100 \cdot 20} = 45 \]

The \( P = 45 \) may be a compromise between what is optimal for 100 ton to 20 ton transport + other functions, such as response to pH change.

*Because \[ \frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{g(x) \frac{d}{dx} \left( f(x) \right) + f(x) \frac{d}{dx} \left( g(x) \right)}{(g(x))^2} \]