Answer to Problems: Transmembrane Transport

1. \( K = 1 \). At equilibrium, we expect \([C]_{in} = [C]_{out}\).

2. \( \Delta G^{\circ} = -RT \ln K \); \( \ln K = \ln (1/4) = 0 \), so \( \Delta G^{\circ} = 0 \).
   \( (K = 1 \text{ and } \Delta G^{\circ} = 0 \text{ for simple transport equilibria}) \)

3. \( \Delta G = \Delta G^{\circ} + RT \ln (\frac{[C]_{in}}{[C]_{out}}) \)
   \[ = (8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}})(298 \text{K}) \left[ \ln \left( \frac{2.4 \times 10^{-3} \text{M}}{1.2 \times 10^{-3} \text{M}} \right) \right] - \frac{1 \text{kJ}}{10^3 \text{J}} \]
   \[ = 7.4 \text{ kJ/mol} \]

4. The process is not spontaneous; \( \Delta G > 0 \).

5. \( \Delta G = RT \ln \left( \frac{[\text{Na}^+]_{in}}{[\text{Na}^+]_{out}} \right) = RT \ln \left( \frac{5.0 \times 10^{-3} \text{M}}{1.4 \times 10^{-3} \text{M}} \right) = -8.8 \text{ kJ/mol} \)

6. This process is spontaneous; \( \Delta G < 0 \).

7. This equation is the sum of the equilibrium equations in questions 1 and 5. By Hess’s law, \( \Delta G \) is the sum of the \( \Delta G \)s for the two equations: \( \Delta G = (7.4 \text{ kJ/mol}) + (-8.3 \text{ kJ/mol}) \)
   \[ \Delta G = -0.9 \text{ kJ/mol} \]

   **Note:** You could also calculate \( \Delta G \) this way:

   \[ \Delta G = RT \ln \left( \frac{[C]_{in} [\text{Na}^+]_{in}}{[C]_{out} [\text{Na}^+]_{out}} \right) = -0.9 \text{ kJ/mol} \]

8. Co-transport is spontaneous; \( \Delta G < 0 \). The \( \text{Na}^+ \) gradient drives uptake of \( C \).
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9. \( \Delta G = \frac{RT \ln Q}{n} + nZ \Delta \psi \). For process shown in 7,

\[ \Delta \psi = \psi_{in} - \psi_{out} = -60mV \]

(Change on Na\(^{+}\))

\[ \Delta G = -0.9 \frac{kJ}{mol} + (1)(96,485 \frac{J}{V \cdot mol})(-60 \times 10^{-3} V) \cdot \frac{1 \frac{kJ}{mol}}{10^3 J} \]

\[ = -0.9 \frac{kJ}{mol} - 5.8 \frac{kJ}{mol} = -6.7 \frac{kJ}{mol} \]

10. \( \Delta \psi \) makes co-transport MORE favorable.

(\( \Delta G = -0.9 \frac{kJ}{mol} \) versus \( \Delta G = -6.7 \frac{kJ}{mol} \))

11. \( \Delta G = RT \ln \left( \frac{[\text{Na}^{+}]_{out}}{[\text{Na}^{+}]_{in}} \right) + nZ \Delta \psi \)

\[ \Delta G = 8.3 \frac{kJ}{mol} + 5.8 \frac{kJ}{mol} = 14.1 \frac{kJ}{mol} \]

12. This process IS NOT spontaneous. Sodium ions are moving against the concentration and voltage gradients.

13. Use Hess's law. The overall process is the sum of:

a) \( \text{ATP} + \text{H}_{2}\text{O} \rightleftharpoons \text{ADP} + \text{P}_{i} \) \( \Delta G = -50.0 \frac{kJ}{mol} \)

and b) \( 3\text{Na}^{+}_{in} \rightleftharpoons 3\text{Na}^{+}_{out} \) \( \Delta G = \frac{14.1 \frac{kJ}{mol}}{3} = 4.7 \frac{kJ}{mol} \)

\[ \Delta G_{\text{total}} = -7.7 \frac{kJ}{mol} \]

14. ATP-driven transport of Na\(^{+}\) IS spontaneous.

15. Primary active transport of Na\(^{+}\) maintains ion and voltage gradients that support secondary active transport of C into intestinal cells.